Similarity between Flashing and Nonflashing Two-Phase Flows

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The purpose of this note is to show that, under certain simplifying assumptions, the generalized solutions developed recently for homogeneous-equilibrium flashing choked flow (Leung, 1986; Leung and Grolmes, 1987) can be extended to the nonflashing flow case by simply redefining the dimensionless parameter known as ω in the final equations.

This ω parameter has been shown in these earlier works to successfully correlate flashing choked flow of a wide range of inlet conditions and fluid properties. Although flashing flow and nonflashing flow differ in one major aspect—the presence of phase change (as a result of pressure drop) in the former and absence of such a phenomenon in the latter, the governing conservation equations for evaluating their respective discharges through nozzles and pipes do not differ in form. Based on a theoretical approach in deriving a more general dimensionless parameter, a unified treatment is shown to be possible for these compressible flow processes.

Correlating Parameter in Compressible Flow

Consider the compressibility of a two-phase flow mixture in terms of its property group dv/dP, i.e., the change in specific volume with respect to change in pressure. Its significance lies in the fact that the critical mass velocity is determined solely by this derivative evaluated at the choking location along a constant entropy path, i.e.,

$$G^2 = \frac{-1}{(dv/dP)_s} \tag{1}$$

For a homogeneous two-phase flow mixture, this derivative takes on the form,

$$\frac{dv}{dP} = \frac{d}{dP}[xv_v + (1-x)v_\ell] = x\frac{dv_v}{dP} + v_{v\ell}\frac{dx}{dP} \tag{2}$$

To estimate the relative magnitude of these two terms in Eq. 2, we assume the flashing process to be isenthalpic (constant enthalpy path) and in thermodynamic equilibrium, yielding

$$\frac{dx}{dT} = -\frac{C_{p\ell}}{h_{v\ell}} \tag{3}$$

Invoking the Clapeyron relation, Eq. 2 can be written as

$$\frac{dv}{dP} = x\frac{dv_v}{dP} - C_{p\ell}T\left(\frac{v_{v\ell}}{h_{v\ell}}\right)^2 \tag{4}$$

Note that for the case of nonflashing flow, the quality or vapor mass fraction x is a constant and the second term hence vanishes in Eq. 4. By further assuming a constant temperature expansion process (usually a good approximation for two-phase flow since the condensed phase possesses far more sensible heat than the vapor phase) with ideal gas behavior, the above equation can be integrated with respect to pressure from the upstream condition, i.e.,

$$\int_{v_o}^{v} dv = P_o v_{vo}\int_{P_o}^{P} -\frac{x\,dP}{P^2} + C_{p\ell}T_oP_o^2\left(\frac{v_{v\ell o}}{h_{v\ell o}}\right)^2\int_{P_o}^{P} -\frac{dP}{P^2} \tag{5}$$

In arriving at the above result, $h_{v\ell}$ has been assumed constant (same as the stagnation value $h_{v\ell o}$) and $Pv_{v\ell} \approx P_o v_{v\ell o}$ as in isothermal gas flow relation (with $v_{v\ell} \approx v_v$ unless near the thermodynamic critical point). We shall make another approximation regarding x inside the first integral. It can be easily shown that the first term in Eq. 5 becomes important only when x is close to unity (i.e., the high-quality flow regime), but in this region x is nearly invariant with change in pressure. Hence, x can be regarded as a constant and be approximated by x_o, the stagnation quality. Upon integration and rearranging, Eq. 5

becomes

$$\frac{\frac{v}{v_o} - 1}{\frac{P_o}{P} - 1} = \frac{x_o v_{vo}}{v_o} + \frac{C_{p\ell} T_o P_o}{v_o}\left(\frac{v_{v\ell o}}{h_{v\ell o}}\right)^2 \tag{6}$$

Note that the two terms on the righthand side are entirely in terms of stagnation properties and, as such, this expression represents a useful approximation for the expansion law or the equation of state (EOS) of a flashing two-phase flow mixture. Unlike the earlier derivation (Leung, 1986), no assumption was made in regard to the form of the EOS. Thus for a *flashing mixture*, the key dimensionless parameter is

$$\omega = \frac{x_o v_{vo}}{v_o} + \frac{C_{p\ell} T_o P_o}{v_o}\left(\frac{v_{v\ell o}}{h_{v\ell o}}\right)^2 = \alpha_o + \rho_o C_{p\ell} T_o P_o \left(\frac{v_{v\ell o}}{h_{v\ell o}}\right)^2 \tag{7}$$

(In the earlier work, the first term was given as $x_o v_{v\ell o}/v_o$; the difference is hardly noticeable for $v_{v\ell o} \approx v_{vo}$.) Whereas for a *nonflashing mixture*, with dx/dP being zero in Eq. 2, the second term in Eqs. 5 and 6 vanishes also, hence

$$\omega = \alpha_o \tag{8}$$

In general, the parameter ω is made up of two distinct terms: the first reflects the compressibility of the mixture due to the existing void fraction (α_o), and the second reflects the compressibility due to phase change or flashing. It can be shown numerically that the "compressibility" of a flashing mixture in terms of the ω value is much greater than that of a nonflashing mixture for the same upstream conditions.

Simplified Expansion Law

Equation 6 suggests a general expansion law for a two-phase mixture, *viz.*,

$$\frac{v}{v_o} = \omega\left(\frac{P_o}{P} - 1\right) + 1 \tag{9}$$

which is applicable for both flashing and nonflashing flows. This form of EOS was used in previous works to obtain the generalized solutions for flashing flow through nozzles and long pipes. To extend these solutions to the nonflashing two-phase flow case, we simply define the ω parameter to be equal to the inlet void fraction α_o.

Critical Discharge through Nozzle

The simplified solution for critical discharge through a perfect nozzle in equation form has been given previously (see Eqs. 5 and 10 of Leung, 1986). The complete solution can now be presented in Figure 1, where both flashing and nonflashing flow results are clearly illustrated in terms of their normalized mass velocities and critical pressure ratios. The boundary between these two regimes is the all-vapor flow condition as denoted by $\omega = 1$. Note that both $G/\sqrt{P_o \rho_o}$ and P_c/P_o attain a value of 0.606 in this case, which would correspond to a fictitious gas having a specific heat ratio γ of 1.0 or an isothermal flow approximation.

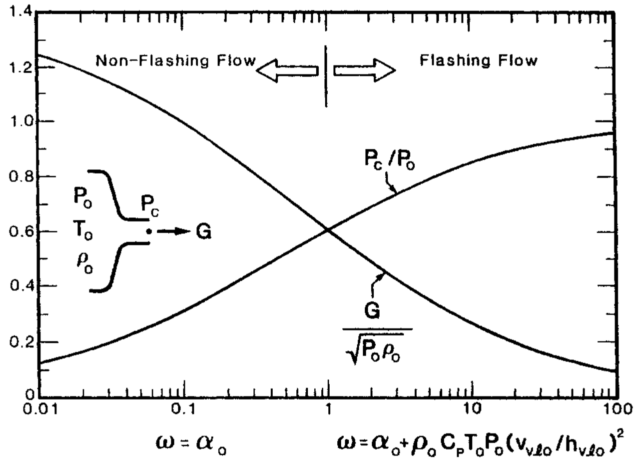

Figure 1. Generalized choked flow solutions for frictionless nozzle discharge.

Critical Discharge through Pipe

Likewise, the complete solution for critical discharge from a horizontal pipe can be illustrated by Figure 2 (for the solution in equation form, see Eqs. 8–10 of Leung, 1987). Here the flow reduction factor, G/G_o, which is the ratio of the mass velocity in the pipe to the mass velocity in a nozzle, is plotted as a function of the pipe resistance factor $4f\,L/D$ with ω as the correlating parameter. It is to be noted that G/G_o curves for nonflashing flow all lie below the flashing flow curves and that the solution for $\omega = 1$ (all-vapor flow) is simply the isothermal flow approximation as applied throughout the pipe.

The solution for critical discharge from an inclined pipe can be correlated by introducing a so-called "flow inclination" parameter (Leung and Epstein, 1990a), defined as

$$Fi = \frac{g\,D\cos\theta}{4f\,P_o\,v_o} = \frac{\rho_o\,g\,L\cos\theta}{(4f\,L/D)\,P_o} \tag{10}$$

where θ is the angle of inclination with the vertical. For a given pipe resistance factor, the parameter Fi represents the ratio of the potential energy to the flow energy and is a measure of the

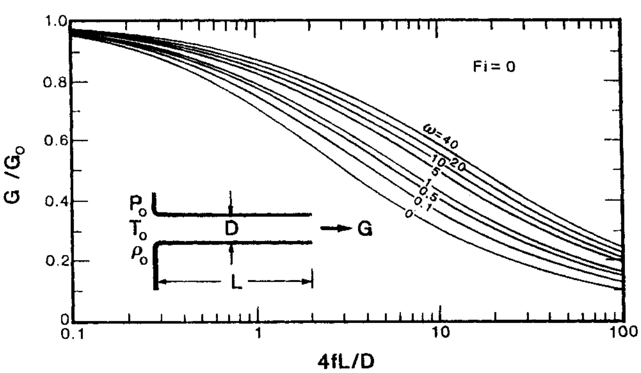

Figure 2. Generalized choked flow solutions for horizontal pipe discharge.

departure from the horizontal case ($Fi = 0$). Figure 3 illustrates the complete solution for the entire ω range at a Fi value of 0.1 in an upflow configuration. Note that for similar ω values the G/G_o curves in this orientation are lower than the corresponding curves in the horizontal flow case—the upflow configuration encounters additional pressure drop due to elevation change.

It is worth noting that the limiting solutions corresponding to $\omega = 0$ (or 0.001 as used in the numerical calculations) approach the expected solution for incompressible liquid discharge which, for the case of no back pressure, can be given as

$$\frac{G}{G_o} = \left[\frac{1 - \left(4f\frac{L}{D}\right)Fi}{1 + 4f\frac{L}{D}}\right]^{1/2} \tag{11}$$

Thus for Figure 2 (with $Fi = 0$), the $\omega = 0$ curve is in excellent agreement with $G/G_o = 1/\sqrt{1 + 4fL/D}$. As for the upflow case with $Fi = 0.1$, the generalized solution in Figure 3 yields a no-flow condition at a $4fL/D$ value of 10, which is in perfect agreement with Eq. 11.

Accuracy and Limitations

The present solutions are limited to homogeneous two-phase flow in frictionless nozzles and in pipes with a constant friction factor. In an adiabatic flashing process through a nozzle, the present solution based on isenthalpic equilibrium flow approximation is almost indistinguishable from an isentropic calculation, particularly for low-quality two-phase flow. This was supported by the excellent agreement between the present nozzle flow solution and the exact isentropic calculation for a wide range of ω values (see Leung, 1986). Deviation was observed only when ω was less than 2, corresponding to the high-quality flow region. In the limit when $\omega = 1$, the all-vapor flow case, the isenthalpic assumption reduces to an isothermal process. It is at this vapor flow condition where the greatest discrepancy is expected between the exact solution and the current approximation. For steam flow, the isothermal solution underpredicts G by about 10%.

As for the nonflashing flow regime, the isothermal approximation as employed here is almost indistinguishable from the thermal equilibrium assumption. This is because the condensed liquid-phase possesses far more thermal (heat) capacity than the vapor phase; thus it can easily supply heat to the expanding vapor to keep the flow process essentially at constant temperature. The so-called two-phase isentropic exponent Γ for a thermal equilibrium process, as first derived by Tangren et al. (1949), is given as

$$\Gamma = \frac{x\, C_{pg} + (1 - x) C_{p\ell}}{x\, C_{vg} + (1 - x) C_{p\ell}} \tag{12}$$

where for $x \ll 1.0$, Γ is nearly 1.0, thus approaching the isothermal limit. The other limiting solution is the frozen model (thermally insulated or no heat transfer between the two phases) where the isentropic exponent is solely that of the gas value $\gamma = C_{pg}/C_{vg}$. Real processes are expected to lie between these two limits. The exact solutions as presented recently (Leung and Epstein, 1990b) show that the isothermal flow solution ($\Gamma = 1.0$) would give a lower G value than the frozen flow solution, and like the flashing flow case, the maximum deviation between these two cases occurs at $\alpha_o = 1.0$. It can be shown that the present nonflashing solution for nozzle flow is in perfect agreement with the isothermal solution presented by Tangren et al. (1949).

Likewise, with the presence of the liquid phase, the isenthalpic assumption in flashing flow and the isothermal assumption for nonflashing flow provide close approximation of the adiabatic frictional flow process in pipes. Again, the largest discrepancy occurs at the all-vapor flow condition—current solution is low by about 15%. Even in this case it is common practice to employ the isothermal-limit (or fictitious gas with $\gamma = 1.0$) solution in pipe design (see Coulson and Richardson, 1954; API, 1976; Zappe, 1981).

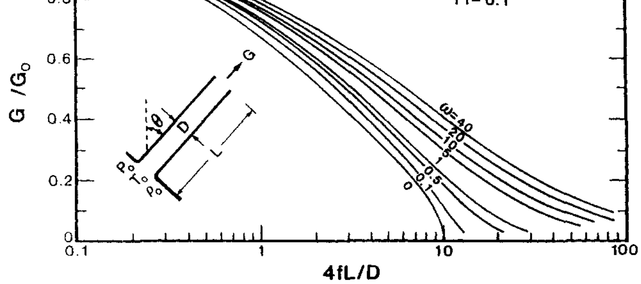

Figure 3. Generalized choked flow solutions for inclined pipe discharge with $Fi = 0.1$.

Notation

C_p = specific heat at constant pressure
C_v = specific heat at constant volume
D = hydraulic diameter of duct
f = Fanning friction factor
Fi = flow inclination number
g = gravitational constant
G = critical mass velocity
$h_{v\ell}$ = latent heat of vaporization
L = duct length
P = pressure
P_c = critical or choked pressure
T = temperature
v = two-phase specific volume = $xv_v + (1 - x)v_\ell$
v_ℓ = liquid specific volume
v_v = vapor specific volume
x = quality or gas mass fraction

Greek letters

α_o = inlet void fraction
ω = parameter as defined by Eq. 7
ρ_o = inlet density
θ = angle of inclination with the vertical
γ = gas-phase specific heat ratio
Γ = two-phase isentropic exponent as defined by Eq. 12

Subscripts

g = gas
ℓ = liquid
o = stagnation condition
s = constant entropy path
v = vapor
$v\ell$ = difference between vapor and liquid properties

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